Employer cette base pour produire une TRÈS courte présentation sur l'intérêt des estimé
énergétique
Fermi problem ou Estimation de Fermi, problème original du piano,wiki
Combien d'enfants sur terre sont actuellement en train de rire à faire couler du lait par leu
Refaire (réemployer) le calcul de puissance consommée par l'humanité.
Consommation de fuel
Biofuel from waste.
Energy storage
Changement climatique
REF: https://www.its.caltech.edu/~oom/
http://www.inference.org.uk/sanjoy/oom/1997/
Livre de Goldreich, Mahajan, Phinney:
What is the annual cost of lighting the streets of Pasadena (Montreal), California?
1M$! 100 000$/an ok, 10M$/an ok mais pas 10 000$ ni 100M$/an.
Peut-on faire mieux?

The Power of Physics Estimation

Tom Murphy

UCSD Physics/CASS

Inspired By…

- Famous physicists like Fermi and Feynman frequently formulated fantastic feats of estimation
  - optional: “estimation” $\rightarrow$ “finagling figures”
- Best course I ever took: Order of Magnitude Physics at Caltech
  - team-taught by Peter Goldreich and Sterl Phinney
- Estimation and Scaling in Physics (UCSD Phys 239)
  - team-taught by Fuller, Diamond, Murphy spring 2010, spring 2012

Our Trajectory Today

- Fermi problems
- Materials properties
- Some time in the clouds
- Fuel economy of cars
- Energy scales (biofuels, waste, storage)
- Climate Change

Color Coding to Clarify

- Black: generic
- *Orange-brown italics: emphasis*
- Red: assumptions
- Blue: constants/knowledge
- Purple: results

- A note on numbers:
  - $\pi = 3 = \text{sqrt}(10) = 10/3$
  - $2 \neq 3$, but $8 \approx 9$
  - c, e, h, k_B, m_p, m_e, σ, G, N_A, μ_0, ε_0, R_E, M_E, r_{AU}, etc. by memory

Fermi Problems

- How many *piano tuners* in Chicago?
- How many molecules from *Julius Caesar's last breath* do you draw in on each breath?
- How far does a car travel before a *one-molecule layer* is worn from the tire?
- How many *laser pointers* would it take to visibly illuminate the *Moon*?
- How heavy is a typical *cloud*?

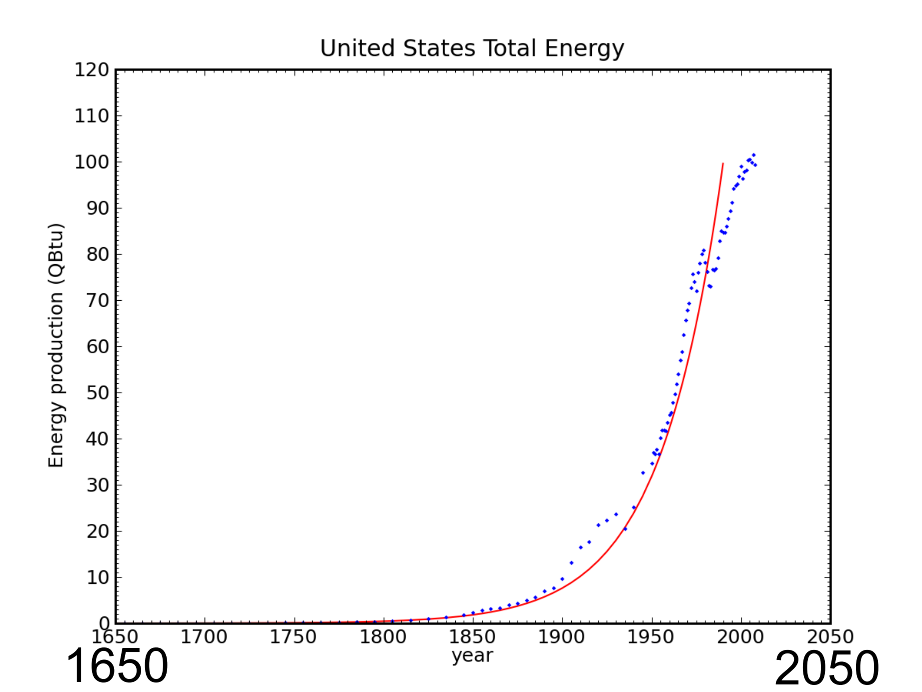
- Book: Guesstimation (by Weinstein and Adam)

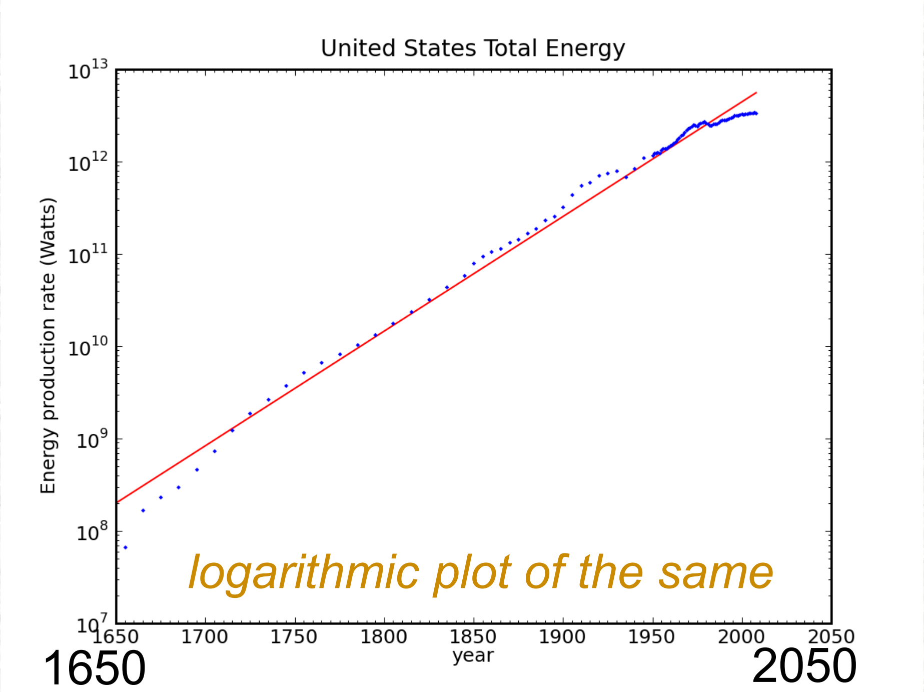
Example Fermi Problem

- How many kids are *laughing so hard* right now that milk (or cultural equivalent) is streaming out of their noses?

- 7 billion people in world
- life expectancy: 60 years
- vulnerable age: 4 to 10 $\rightarrow$ 10% of life $\rightarrow$ 700 million at risk
- half of people have had this experience $\rightarrow$ 350 M at risk
- once-in-lifetime event, 10 sec duration $\rightarrow$ $10/(6 \times \pi \times 10^7)$
- 350 M $\times$ $0.5 \times 10^{-7} \approx 20$

Fermi Approach Applied to Exponentials



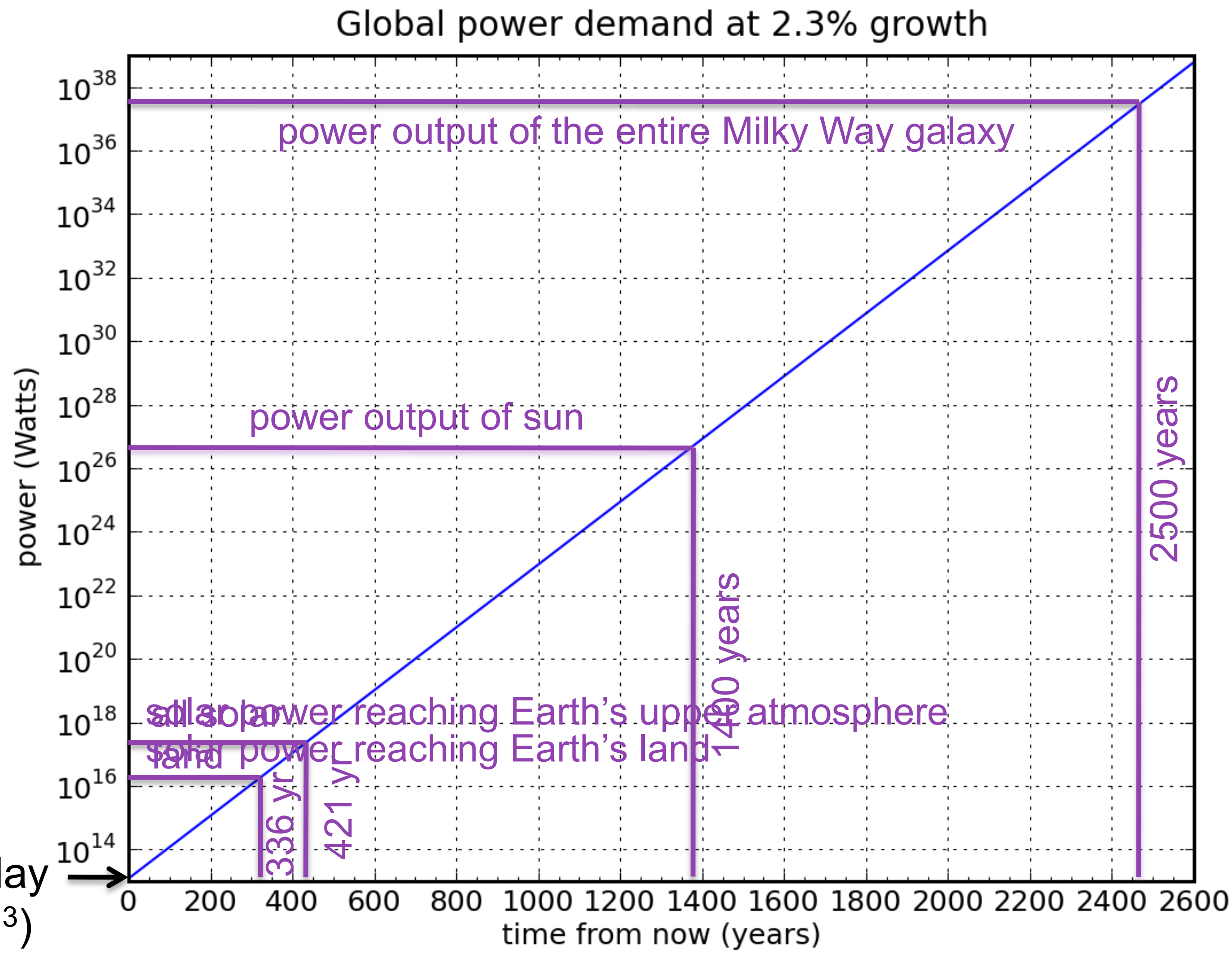


Sum of all forms of energy used in the U.S. (fossil fuels, nuclear, hydro, wood, etc.)

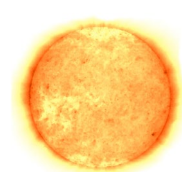
Red curve is exponential at 2.9% per year growth rate

World is at 12 TW now; pick 2.3% rate, mapping to 10 × per 100 yrs.

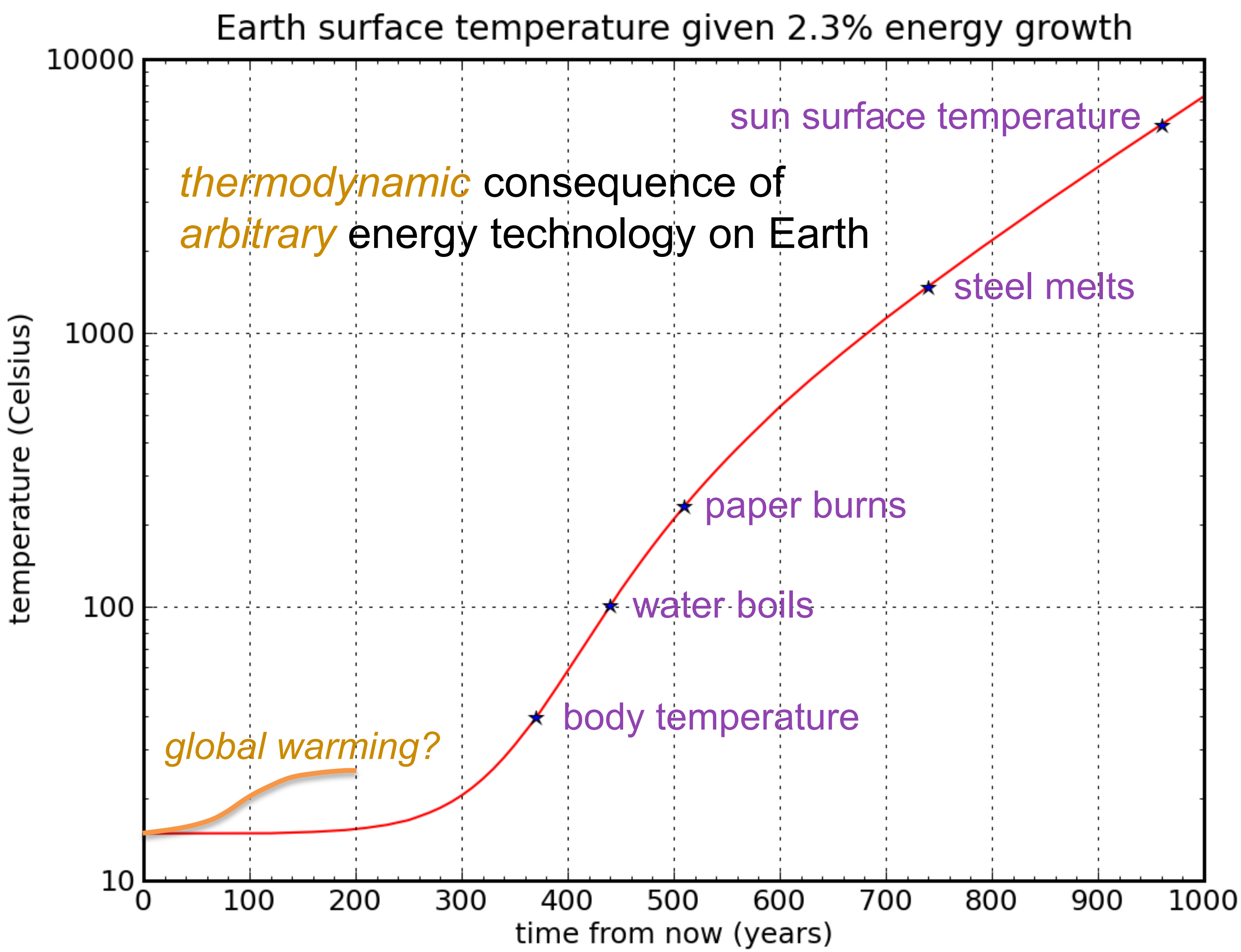
Extrapolating at 10 × per Century



12 TW today
(1.2×10^{13})



Waste Heat Boils Planet (not Global Warming)



Straightforward application of σT^4 radiative disposal of heat

Materials Properties

- *Heat Capacity*
- Thermal Conductivity
- *Strength of materials*
- Thermal Expansion

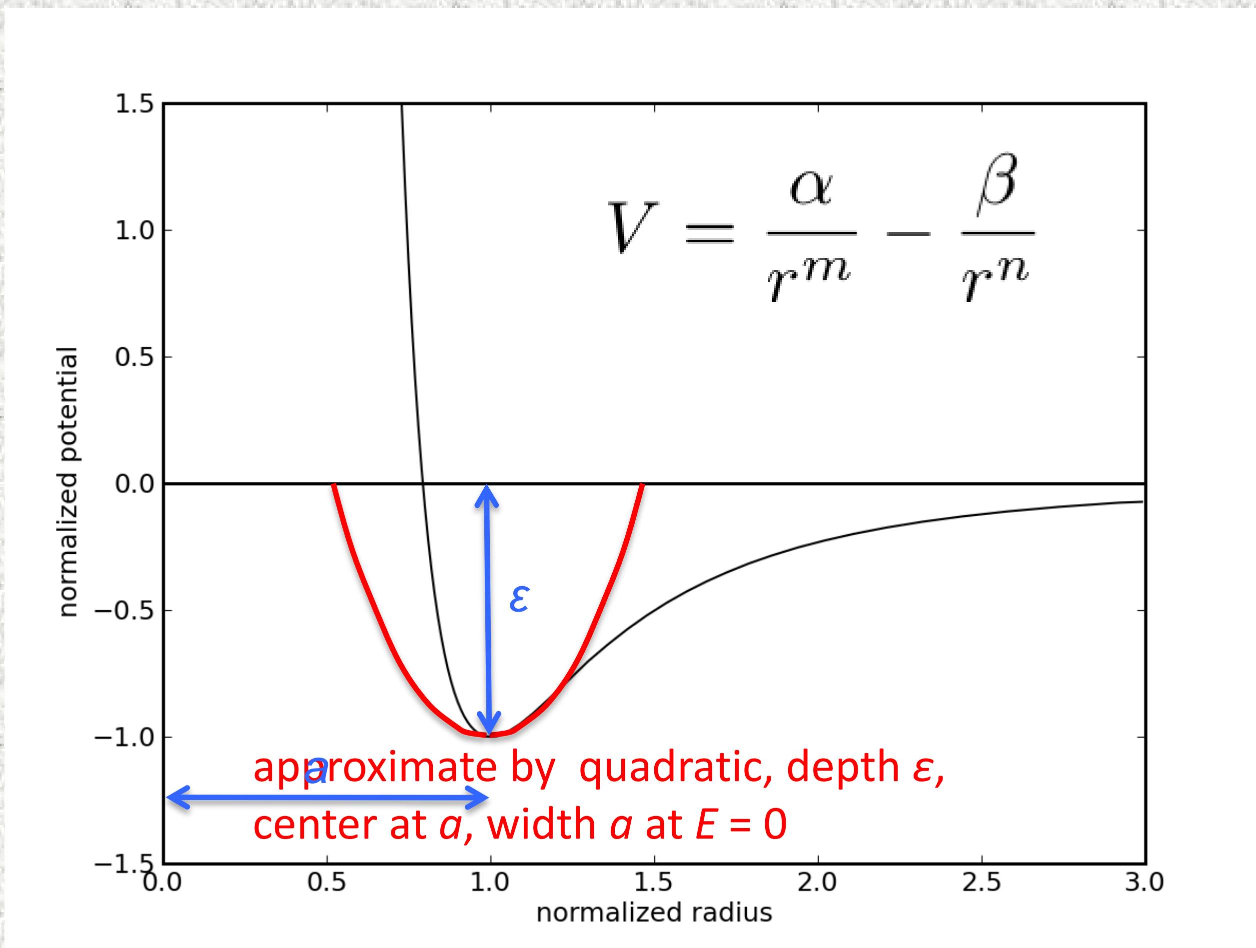
- All from knowledge of bond strength (eV scale), atomic number, density, kT at room temperature (1/40 eV)

Heat Capacity

- $3/2\ kT$ per particle
- derivative is just $3k/2$, Joules per Kelvin per particle
- Want J/K/kg
- 1 kg has $1000N_A/A$ particles
- $c_p = 1500 \times N_A \times k/A \approx 12000/A$ J/K/kg
  - Note $N_A \times k = 6 \times 10^{23} \times 1.4 \times 10^{-23} \approx 8$ (R = 8.3 J/mol/K ideal gas constant)
- Since most of our world has $A \approx 10–50$, $c_p \approx 200–1000$ J/K/kg

- Can get thermal conductivity for gas using mean-free-path and relating to diffusion equation

Mechanics of Solids: Potential



$$V = \frac{\varepsilon}{m-n}\left[n\left(\frac{a}{r}\right)^m - m\left(\frac{a}{r}\right)^n\right]$$

$$V = 4\varepsilon\left(\frac{r-a}{a}\right)^2 - \varepsilon$$

Getting the Elastic Modulus

$$V = 4\varepsilon\left(\frac{r-a}{a}\right)^2 - \varepsilon \;\rightarrow\; F = -\frac{dV}{dr} = -\frac{8\varepsilon}{a}\left(\frac{r-a}{a}\right) = -\frac{8\varepsilon}{a^2}\delta r$$

- Associate *spring constant* with $8\varepsilon/a^2$
- Have one spring per area a^2, so stress (force per area) is

$$\sigma = \frac{F}{A} = \frac{8\varepsilon}{a^4}\delta r = \frac{8\varepsilon}{a^3}\frac{\delta r}{a} = E\epsilon$$

- Associate *elastic modulus*, E, with $8\varepsilon/a^3$
- For $\varepsilon \approx 1$ eV = 1.6×10^{-19} J; $a \approx 2$ Å (2×10^{-10} m)
  - can get a from density and atomic number
- $E \approx (8 \times 1.6 \times 10^{-19})/(8 \times 10^{-30}) = 160 \times 10^9$ Pa (160 GPa)
  - right in line with many materials

Drop a Coffee Mug: how many pieces?

- Model as cylinder 0.1 m by 0.1 m, t = 0.005 m wall thickness
- *Volume* 0.3 × 0.1 × 0.005 = 1.5 × 10^{-4} m^3; 2000 kg/m^3 → 0.3 kg
- From 1 meter, 3 J of energy
- f = 10% goes to breaking bonds (W = 0.3 J)
  - the rest to heat in ringing pieces
  - kinetic energy of pieces
- Number of bonds broken: W/ε
- Area per bond ≈ a^2
- Area of fractured zone: Wa^2/ε
  - $A \approx (0.3 \times 4 \times 10^{-20})/(1.6^{-19}) \approx 7.5 \times 10^{-2}$ m^2
  - fracture length, $L = A/t$ = 15 meters

Coffee Mug, Part 2

- Have fracture Length, L; say it breaks into N square chunks, side length ℓ
- Each square has 2ℓ length of unique breakage it can claim
  - don't want to double-count
- $2N\ell = L \approx 15$ m
- Total mug area is $N\ell^2 = (0.3 \text{ m}) \times (0.1 \text{ m}) = 0.03 \text{ m}^2$
- Solve for $\ell = (0.03 \text{ m}^2)/(7.5 \text{ m}) = 0.004 \text{ m} \rightarrow 4$ mm
- $N \approx 2000$

$$N = \frac{f^2 \rho^2 V g^2 h^2 a^4}{4\varepsilon^2 t}$$

Cloud Computing

- *How much does a cloud weigh*?
- Nice illustration of multiple techniques/angles often possible in attacking physics problems
- Will work on two aspects:
  - (over) *density* of clouds
  - *droplet size*

Giant Thunderstorm

- Imagine a towering cumulonimbus, 10 km tall (30,000 ft) dumps all of its water
- Expect you'll record something like 1–10 inches of rain
  - let's say 0.1 m
- Each square meter has 100 kg (cubic meter is 1000 kg)
- In 10 km cloud column: (100 kg)/(10,000 m^3) = 0.01 kg/m^3
  - about 1% of air density

Bumpy Ride

- Airplanes fly into clouds all the time
- Sometimes bumpy due to turbulent convection
- But *no noticeable horizontal deceleration* on hitting the wall
- Drag force goes like $\frac{1}{2}\rho c_D A v^2$, where *ρ is density of medium*
- Drag force is about 5% of lift force (picture aerodynamic flow)
- If cloud density were 10% that of air, drag would surge by 10%
  - would correspond to 0.5% g
  - sudden onset would be very noticeable
- So cloud density << 10% air density

- Lift also proportional to density, so vertical more sensitive

Saturation Pressure

- Gas phase occupies 22 liters/mole at STP
  - but vapor pressure exponentially suppressed at temperatures below boiling point (Maxwell-Boltzmann tail)
  - another view: 100° C saturation pressure is 760 Torr; 20° C → 17.5 Torr
  - results in density ratio (17.5/760) × (18/29) = 1.5%
  - less than this at actual temperatures at base of cloud (where condensation begins)
- Can go through *order-of-magnitude* process too
  - balance rates of entry/exit at liquid/vapor interface using Maxwell-Boltzmann tail

Droplet Size from Terminal Velocity

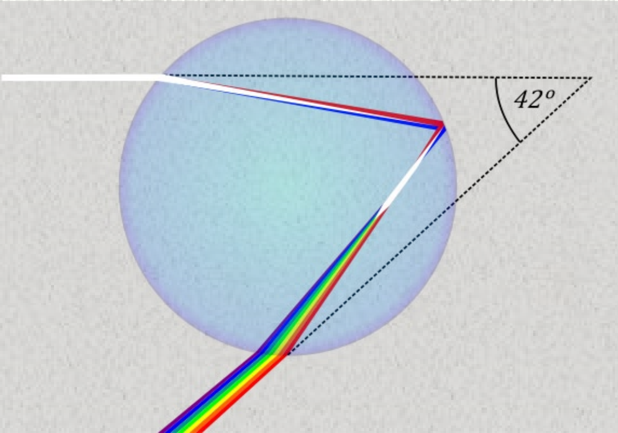
- Particles must be small enough that terminal velocity is very small
  - pick 10 cm/s (easily overcome by air currents)
- Stokes drag regime: $F_d = 6\pi\rho_a \nu r v$
  - 6π is an *enemy* of the order-of-magnitude scaling approach
- r and v are droplet radius and velocity; ρ_a and ν are density and kinematic viscosity ($\approx 10^{-5}$ m^2/s for air)
- Set equal to $mg = 4\rho_w r^3 g$ to get r:
  - $r^2 = 1.5\pi(\rho_a/\rho_w)(\nu v/g) \approx 6 \times 10^{-3} \times 10^{-5} \times 10^{-1}/10 = 6 \times 10^{-10}$ m^2
  - $r \approx 25$ microns
- Check *Reynolds number*: $Re = rv/\nu \approx (10^{-5} \times 10^{-2})/10^{-5} = 10^{-2}$
  - safely under 1, so in Stokes (viscous) regime

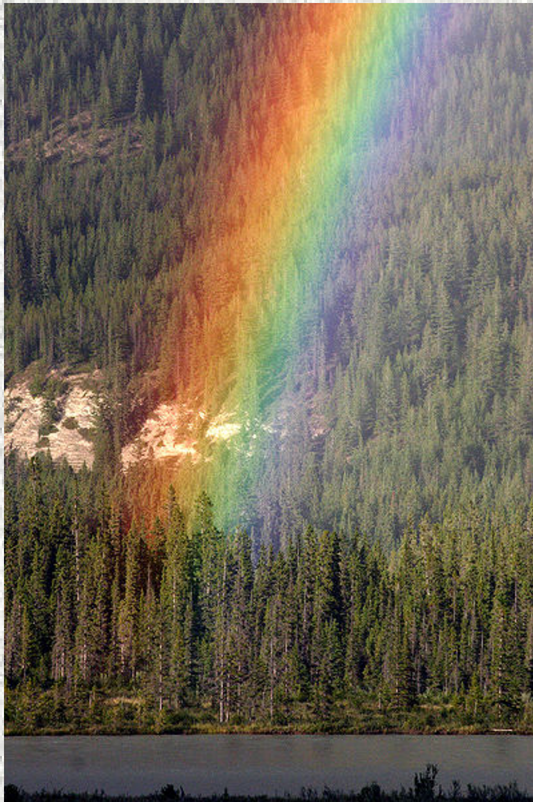
Droplet Size from Optical Depth of Fog

- Flying in cloud (or driving in heavy fog), might have a 5 m limit to line of sight
- mean-free path: $\lambda = 1/n\sigma$
  - n is space density, σ is cross section (πr^2)
  - using 1% air density, $\rho_c = 4\rho_w r^3 n \approx 0.01$ kg/m^3 $\rightarrow n = ¼(\rho_c/\rho_w)r^{-3}$
- Putting pieces together, $r \approx \lambda(\rho_c/\rho_w) \approx 5 \times 10^{-5}$ m
  - 50 microns

Droplet Size Inferred from Rainbows

- We see rainbows when rain drops are present, but not against clouds
- Why not? Still spherical droplets with refractive dispersion
  - the same geometry works
- Problem is diffraction: λ/D is too small
  - washes out pattern
- Rainbow width is about 1°, or 0.017 radians
  - need $\lambda/D \gg 0.02$ to wash out pattern
  - $D \ll 50\lambda \approx 25\ \mu m$





Multiple Approaches Penetrate the Fog

- The cloud examples illustrate the value of *multiple approaches*
  - corroborate understanding
- Bring to bear loads of *common-sense observations*
  - many of us already know these things, even if we didn’t think we did
- Helps to ask yourself what range of direct experiences you have with the matter at hand
  - *what handles* can you invent?

Is 100 MPG from gasoline possible?

- At freeway speeds, mainly fight drag: $F_d = \frac{1}{2}\rho c_D A v^2$
  - $\rho = 1.2$ kg/m^3, $c_D \approx 0.3$, $A \approx 2.5$ m^2, $v = 30$ m/s
  - $F_d \approx 400$ N
- Rolling resistance is about $0.01mg \approx 100$ N (indep. of v)
- Net 500 N
- A gallon of gasoline (3 kg × 10 kcal/g × 4.18 kJ/kcal) contains about 130 MJ of energy
- Used at ~25% efficiency in internal combustion engine
- $W = F \times d \rightarrow d = 30$ MJ / 500 N = 60 km ≈ 35 miles
- 100 MPG from gasoline at freeway speeds is *super-hard*
  - need a *factor of four* improvement in drag piece, for instance

Corn Ethanol Or Bust

- Let's calculate how much land we need to replace oil
  - an Iowa cornfield is 1.5% efficient at turning incident sunlight into stored chemical energy
  - the conversion to ethanol is *at best* 30% efficient
    - assuming 1.4:1 ratio, and using corn ethanol to power farm equipment and ethanol production itself
  - growing season is only part of year (say 50%)
  - net is 0.23% efficient (1.5% $\times$ 30% $\times$ 50%)
  - need 40% of 10^{20} J per year = 4×10^{19} J/yr to replace petroleum
  - this is 1.3×10^{12} W: thus need 6×10^{14} W input (at 0.23%)
  - 350 W/m^2 summer insolation, need 2×10^{12} m^2, or (1,400 km)^2 of land
  - that's a square 1,400 km on a side; as a ***lower limit***

What does this amount of land look like?

We don't *have* this much arable land!
And where do we grow our food?

Wasted Energy?

- A recent article at *PhysOrg* touted a methane reclamation scheme from sewage in the L.A. area
- Quotes from within article:
  - *"This is a paradigm shift. We'll be truly fuel-independent and no longer held hostage by other countries. This is the epitome of sustainability, where we're taking an endless stream of human waste and transforming it to transportation fuel and electricity. This is the first time this has ever been done."*
  - *"a third of all cars on the road in the U.S. could eventually be powered by 'biogas,' made from human waste, plant products and other renewable elements."*

Do the Math

- Human metabolism is about 2000 kcal/day ≈ 100 W
- We're pretty good at extracting metabolic energy from food
  - let's be generous and say we forfeit as much as 10% in our poop
  - that's 10 W per person
- In the U.S., we each consume 10,000 W of continuous energy
  - 40%, or 4,000 W is from oil
  - 60% of this, or 2,400 W, is imported
- So we could ***at most*** expect to replace 0.4% of our foreign oil by powering our cars with human waste

Energy Storage

- A major transition away from fossil fuels to solar, wind, etc. will *require massive storage solutions*
- The cheapest go-to solution for stand-alone systems has been lead-acid batteries
  - but national battery would be a cubic mile, and require more lead than is estimated to exist in global *resources* (let alone proven *reserves*)
- We can use estimation techniques to evaluate possible solutions
  - focus on *home-scale solutions*
  - scale will be 100 kWh of storage (3 days elec. for average American)
  - explore gravitational, batteries, compressed air, flywheels

Gravitational Storage

- Hoisting rocks or pumping tanks of water: low tech approach
- A rechargeable AA battery (1.5 V, 2 A-h $\rightarrow$ 3 Wh ≈ 10 kJ)
- Hoisting mass on 3 m derrick: need 300 kg to match AA battery
  - gravitational storage is incredibly weak
- 100 kWh, in menacing 10 m high water tower, needs 3600 m^3
  - 15 meters on a side
  - oops

Lead-Acid Batteries

- Each reaction involves a Pb atom in the anode, a PbO_2 molecule in the cathode, and two electrons at 2 eV each
- 100 kWh (3.6×10^8 J) needs 10^{27} Pb atoms
  - 1700 moles; 355 kg of lead; might guess 4 × realistic
  - real batteries would have 1500 kg of lead (2500 kg total battery mass)
- 2500 kg at 2.5 × density of water → 1 cubic meter
  - will cost $15,000
  - actually, the cheapest, most compact of the four we're considering
- For U.S. to go full solar/wind requires significant storage
  - not enough lead in world resources (let alone reserves) to build for U.S.

Compressed Air

- Charged to 200 atm, energy is $P_0 V_0 \ln(P_f/P_0) = 5.3 P_0 V_0$
  - simple integration of $PdV = NkT(dV/V)$
- $P_0 = 10^5$ Pa
- Need $5.3 \times 10^5 V_0 = 100$ kWh $= 3.6 \times 10^8$ J
  - $V_0 = 700$ m^3
  - $V_f = 3.5$ m^3
  - cube 1.5 meters on a side

Flywheel

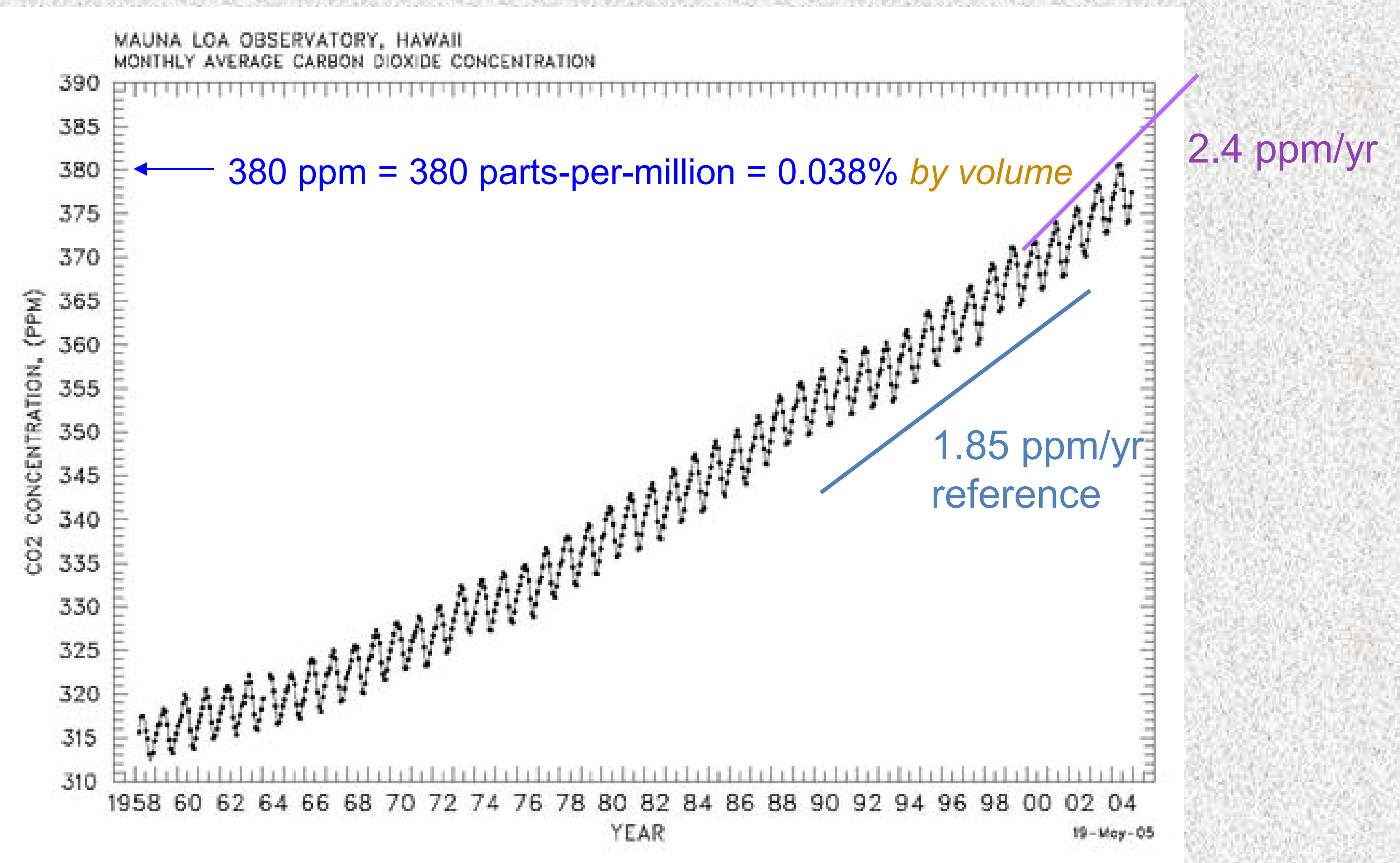
- Solid cylinder: $I = ½MR^2$
- Edge velocity, $v \rightarrow \omega = v/R$; $E = ½I\omega^2 = ¼Mv^2$
- Pick edge velocity $v = 300$ m/s
- Need 16 ton mass
- At density of steel, this is 2 cubic meters
  - e.g., 2 meters high; 1.2 meter diameter
  - acceleration at edge; v^2/R is 16,000g
  - break-up: exceeds mechanical strength
  - need larger, slower to be safe: 2.5 m diameter, 125 m/s
    - 10 m^3; 80 tons $\rightarrow$ 1250g

can get 25 kWh unit 2×3 m; $100k

Heck: Just use a generator!

- Each gallon of gasoline contains 36.6 kWh of thermal energy
- Home Depot generator probably 15% efficient
  - seems like the rest comes out in noise!
  - about 5 kWh of electricity per gallon
- For 100 kWh, need 20 gallons (75 liters) of gasoline
  - *gasoline*: 0.075 m^3
  - *lead acid*: 1.0 m^3
  - *compressed air*: 3.5 m^3
  - *flywheel*: 10 m^3
  - *water/grav at* 10 m: 3600 m^3
- Hard to beat fossil fuels!

The Rise of CO_2



Charles Keeling (SIO), started measuring atmospheric CO_2 from Mauna Loa in Hawaii in 1958. Besides the annual photosynthetic cycle, a profound trend is seen.

Is this rise surprising?

- Every gram of fossil fuel used produces 3 grams of CO_2
  - it's *straight chemistry*: to get the energy out via combustion, the carbon from the hydrocarbon gets attached to oxygen and off it goes
- How much should we expect?
  - global energy budget is 4×10^{20} J/yr; pretend all from fossil fuels
  - average 10 kcal/gram $\rightarrow$ ~40,000 J/gram $\rightarrow$ 10^{16} g/yr F.F.
  - so 3×10^{16} g/yr CO_2 $\rightarrow$ 3×10^{13} kg/yr CO_2
  - atmosphere has mass = 5.3×10^{18} kg $\rightarrow$ CO_2 adds 5.7 ppm/yr *by mass*
  - about 3.7 ppm/yr *by volume* (CO_2 is 44 g/mol vs. 29 for air)
  - 50/50 to ocean/atmosphere, atmospheric rise is 1.85 ppm/yr, *by volume*
  - this is darn close to what we see on the "Keeling curve" graph

Total CO_2 rise

- We can do the same thing for the entire fossil fuel history
  - have gone through 1 trillion barrels of oil → 140 Gtoe
    - Gtoe is gigaton (10^9 ton) oil equivalent (by energy)
  - used about 160 Gtoe coal worldwide
    - using 40 Gtoe U.S. times four, since U.S. uses 25% of world energy
  - used 1037 tcf natural gas in U.S. → 27 Gtoe, so guess 100 Gtoe worldwide
  - 400 Gtoe of fossil fuels → 1.2×10^{15} kg of CO_2 (3× FF mass)
  - 228 ppm of atmosphere *by mass*; 150 ppm *by volume*
  - half into atmosphere → 75 ppm increase
  - see 100 ppm increase (280 ppm pre-industrial to 380 ppm)
- So the CO_2 increase is *absolutely expected*!

Expected Temperature Rise

- If you add to the blanket, expect to get warmer
- Applying σT^4 in radiative equilibrium, Earth is 255 K
  - but actual number is 288 K, thanks to 33 K greenhouse effect
- How much warmer?
  - We know that 7°C of the 33° C greenhouse effect is from CO_2
  - Have gone from 280 to 385 ppm (11/8 times as much, or 3/8 increase)
  - This should translate into 7°×3/8 = 21/8 = 2.6°C change
    - but takes some time because oceans are slow to respond, having *enormous heat capacity*
- Should be *NO SURPRISE* that burning loads of fossil fuels makes us warmer
  - not actually hard to understand!

Summary

- We often *know more than we think* about a problem
- Real world problems *don't come with tidy numbers* attached
- *Estimation* and *multiple techniques* often fruitful
- Every congressperson should have an *estimator* on staff
  - and LISTEN to them!