11.0 EXERCICES ÉNERGIE SOLAIRE

Exercice n° 11.3.j : Angle d'incidence solaire 2

Une surface est orientée à 45º par rapport à l'horizontale et pointe à 15º à l'ouest du sud.

QUESTIONS

Question 1 : Quel est l'angle d'incidence du rayonnement direct sur cette surface située à Madison à 10h30 (temps solaire) le 13 février ?

REPONSES

Question 1 : Quel est l'angle d'incidence du rayonnement direct sur cette surface située à Madison à 10h30 (temps solaire) le 13 février ?

Calculate the angle of incidence of beam radiation on a surface located at Madison, Wisconsin, at 10:30 (solar time) on February 13 if the surface is tilted 45° from the horizontal and pointed 15° west of south.

Solution

Under these conditions, $n = 44$, the declination δ from Equation 1.6.1 is $-14°$, the hour angle $\omega = -22.5°$ (15° per hour times 1.5 h before noon), and the surface azimuth angle $\gamma = 15°$. Using a slope $\beta = 45°$ and the latitude ϕ of Madison of 43° N, Equation 1.6.2 is

$$\begin{aligned}\cos\theta = {} & \sin(-14)\sin 43\cos 45 - \sin(-14)\cos 43\sin 45\cos 15 \\ & + \cos(-14)\cos 43\cos 45\cos(-22.5) \\ & + \cos(-14)\sin 43\sin 45\cos 15\cos(-22.5) \\ & + \cos(-14)\sin 45\sin 15\sin(-22.5)\end{aligned}$$

$$\cos\theta = -0.117 + 0.121 + 0.464 + 0.418 - 0.068 = 0.817$$

$$\theta = 35° \qquad ■$$

There are several commonly occurring cases for which Equation 1.6.2 is simplified. For fixed surfaces sloped toward the south or north, that is, with a surface azimuth angle γ of 0° or 180° (a very common situation for fixed flat-plate collectors), the last term drops out.

For vertical surfaces, $\beta = 90°$ and the equation becomes

$$\cos\theta = -\sin\delta\cos\phi\cos\gamma + \cos\delta\sin\phi\cos\gamma\cos\omega + \cos\delta\sin\gamma\sin\omega \qquad (1.6.4)$$

For horizontal surfaces, the angle of incidence is the zenith angle of the sun, θ_z. Its value must be between 0° and 90° when the sun is above the horizon. For this situation, $\beta = 0$, and Equation 1.6.2 becomes

$$\cos\theta_z = \cos\phi\cos\delta\cos\omega + \sin\phi\sin\delta \qquad (1.6.5)$$

The solar azimuth angle γ_s can have values in the range of 180° to −180°. For north or south latitudes between 23.45° and 66.45°, γ_s will be between 90° and −90° for days less than 12 h long; for days with more than 12 h between sunrise and sunset, γ_s will be greater than 90° or less than −90° early and late in the day when the sun is north of the east-west line in the northern hemisphere or south of the east-west line in the southern hemisphere. For tropical latitudes, γ_s can have any value when $\delta - \phi$ is positive in the northern hemisphere or negative in the southern, for example, just before noon at $\phi = 10°$ and $\delta = 20°$, $\gamma_s = -180°$, and just after noon $\gamma_s = +180°$. Thus γ_s is negative when the hour angle is negative and positive when the hour angle is positive. The sign function in Equations 1.6.6 is equal to +1 if ω is positive and is equal to −1 if ω is negative:

$$\gamma_S = \text{sign}(\omega)\left|\cos^{-1}\left(\frac{\cos\theta_z\sin\phi - \sin\delta}{\sin\theta_z\cos\phi}\right)\right| \qquad (1.6.6)$$